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Modified Variational Iteration Method for the Solution of nonlinear Partial Differential Equations

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Abstract-The Variational Iteration Method (VIM) has been shown to solve effectively, easily and accurately a large class of linear and nonlinear problems with approximations converging rapidly to exact solutions.

We present a new Modified Variational Iteration Method (MVIM) for the solution of some partial differential equations of physical significance.

Index Terms : variational iteration method, lagrange multiplier, Taylor's series, partial differential equation, modified variational iteration method, correction functional.

1.INTRODUCTION

Differential equations play a crucial role in applied mathematics and physics. The results of solving these equations can guide authors to know the described process deeply. But at times it is difficult to obtain the exact solutions to these problems. In recent decades, there has been great development in the numerical analysis and exact method of solving partial differential equations. For instance, Adomian's decomposition method, Homotopy perturbation method, parameter expanding method etc.

He (1999, 2000, 2006) developed the variational iteration method for solving linear, nonlinear and boundary value problems. The method was first considered by Inokuti, Sekine and Mura (1978) and fully explored by He. J. H. In this method, the solution is given in an infinite series usually converging to an accurate solution. Olayiwola *etal* (2009) used modified power series method for the solution of systems of differential equations. It is observed that the method solve effectively, easily and accurately a class of linear, nonlinear, ordinary differential equations with approximate solution which converge very rapidly to accurate solution.

In this paper, we present a new modified variational iteration method for the solution of nonlinear partial differential equations.

2. VARIATIONAL ITERATION METHOD

To illustrate the basic concept of the VIM , we consider the following general nonlinear partial differential equation.

$$Lu(x,t)+Ru(x,t)+Nu(x,t)=g(x,t) \qquad (2.1)$$

where L is a linear time derivative operator, R is a linear operator which has partial derivative with respect to x, N is a nonlinear operator and g is an inhomogeneous term. According to VIM, we can construct a correct fractional as follows:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda\left[Lu_n+R\tilde{u}_n+N\tilde{u}_n-g\right]d\tau \qquad (2.2)$$

where λ is a Lagrange multiplier which can be identified optimally via variational iteration method. The subscript n denote the nth approximation, $\tilde{u}_n$ is considered as a restricted variation i.e, $\delta\tilde{u}_n=0$. The

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successive approximation $u_{n+1}, n \geq 0$ of the solution u will be readily obtained upon using the determined Lagrange multiplier and any selective function u_0, consequently, the solution is given by:

$$u = \lim_{n \to \infty} u_n \tag{2.3}$$

In Modified VIM, equation (2.2) becomes:

$$u_0(x,t) = g_0(x) + tg_1(x) + t^2 g_2(x) \tag{2.4}$$

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda \left[Lu_n + R\tilde{u}_n + N\tilde{u}_n - g\right] d\tau \tag{2.5}$$

where $g_2(x)$ can be found by substituting for $u_0(x,t)$ in (2.1) when $t = 0$.

3. APPLICATION OF MVIM

3.1 We consider the nonlinear homogeneous gas dynamics equation Hossein *etal* (2008):

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(u^2) - u(1-u) = 0, \quad 0 \leq x \leq 1, \quad t > 0. \tag{3.1}$$

With the initial conditions

$$u(x,0) = e^{-x}$$

The iteration formula is

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda \left[\frac{\partial u}{\partial \tau} + \frac{1}{2}\frac{\partial}{\partial x}(u_n^2) - u_n(1-u_n)\right] d\tau \tag{3.2}$$

Making (3.2) correction functional stationary, the Lagrange multiplier can be identified as:

$$\lambda = -1 \tag{3.3}$$

Using (2.4) and (2.5) in (3.2)

$$u_1(x,t) = e^{-x} + \tau e^{-x} - \int_0^t \begin{bmatrix} e^{-x} + \frac{1}{2}(-2e^{-2x} - 4\tau e^{-2x} - 2\tau^2 e^{-2x}) \\ -(e^{-x} + \tau e^{-x}) + e^{-2x} + 2\tau e^{-2x} + \tau^2 e^{-2x} \end{bmatrix} d\tau \tag{3.4}$$

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$$= e^{-x} + t\,e^{-x} + \frac{t^2}{2}e^{-x} \tag{3.5}$$

Similarly

$$u_2 = e^{-x} + t\,e^{-x} + \frac{t^2}{2}e^{-x} + \frac{t^3}{6}e^{-x} \tag{3.6}$$

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$$u_n = e^{-x}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \ldots\ldots + \ldots\right) \tag{3.7}$$

It is obvious that a higher number of iteration makes u_n converges to exact solution: e^{t-x}.

Figure 1 shows the graphs of exact solution against x for different time (t). Figure 2 shows the graphs of MVIM solution against x for different time (t) while Figure 3 shows the graphs of exact solution and MVIM solution against x for (3.1).

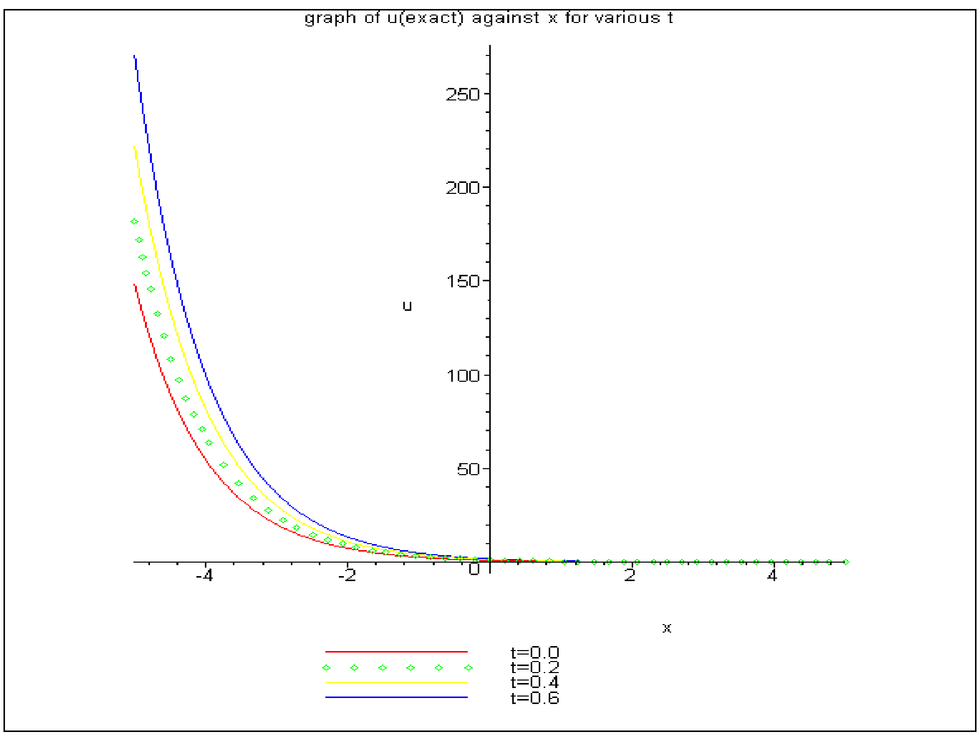

Figure 1

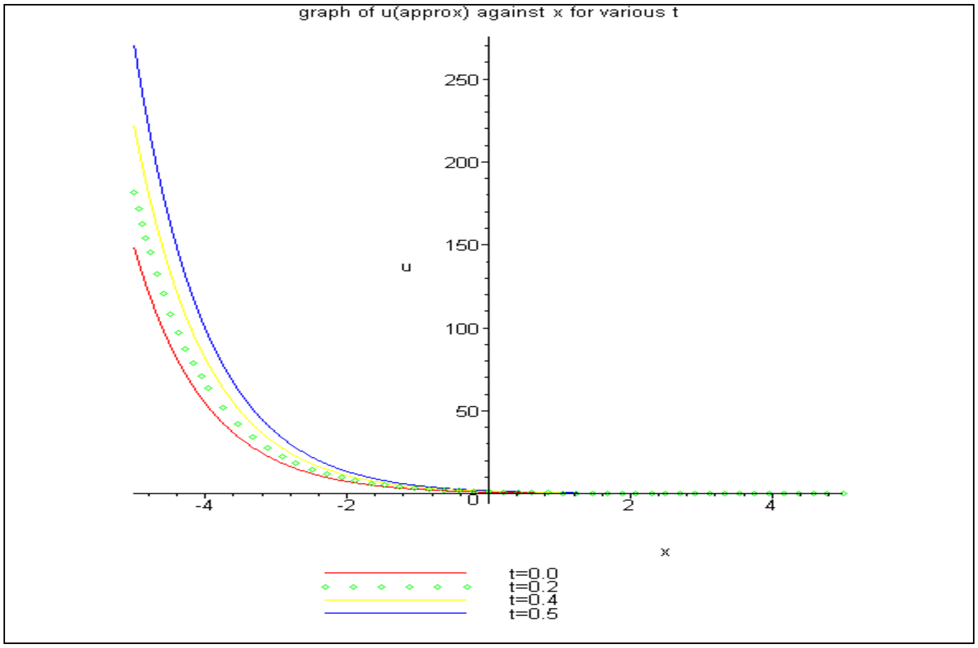

Figure 2

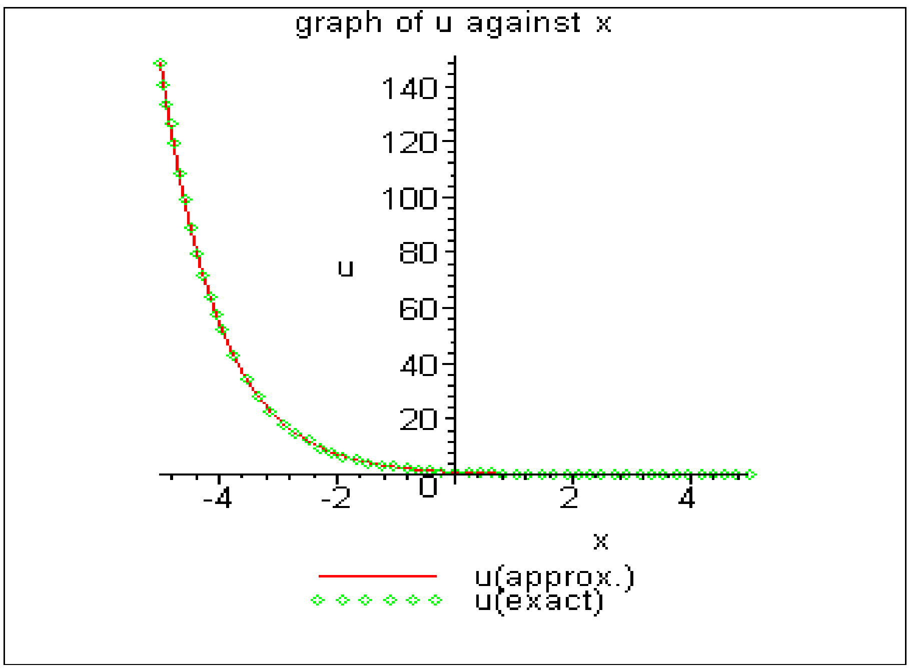

Figure 3

3.2 We consider the nonlinear equation:

$$\frac{\partial u}{\partial t} + u\left(\frac{\partial u}{\partial x}\right)^2 - u^2 \frac{\partial^3 u}{\partial x^3} + \frac{\partial^6 u}{\partial x^6} = 0 . \tag{3.8}$$

$$u(x,0) = e^x$$

Similarly

$$u_0 = e^x - t e^x \tag{3.9}$$

$$u_1 = e^x - t e^x + \frac{t^2}{2} e^x \tag{3.10}$$

$$u_2 = e^x - t e^x + \frac{t^2}{2} e^x - \frac{t^3}{6} e^x \tag{3.11}$$

.
.

$$u_n = e^x \left(1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{24} - .... + ...\right) \tag{3.12}$$

This converges to:

$$u_n = e^{x-t}$$

Figure 4 shows the graphs of exact solution against x for different time (t). Figure 5 shows the graphs of MVIM solution against x for different time (t) while Figure 6 shows the graphs of exact solution and MVIM solution against x for (3.8).

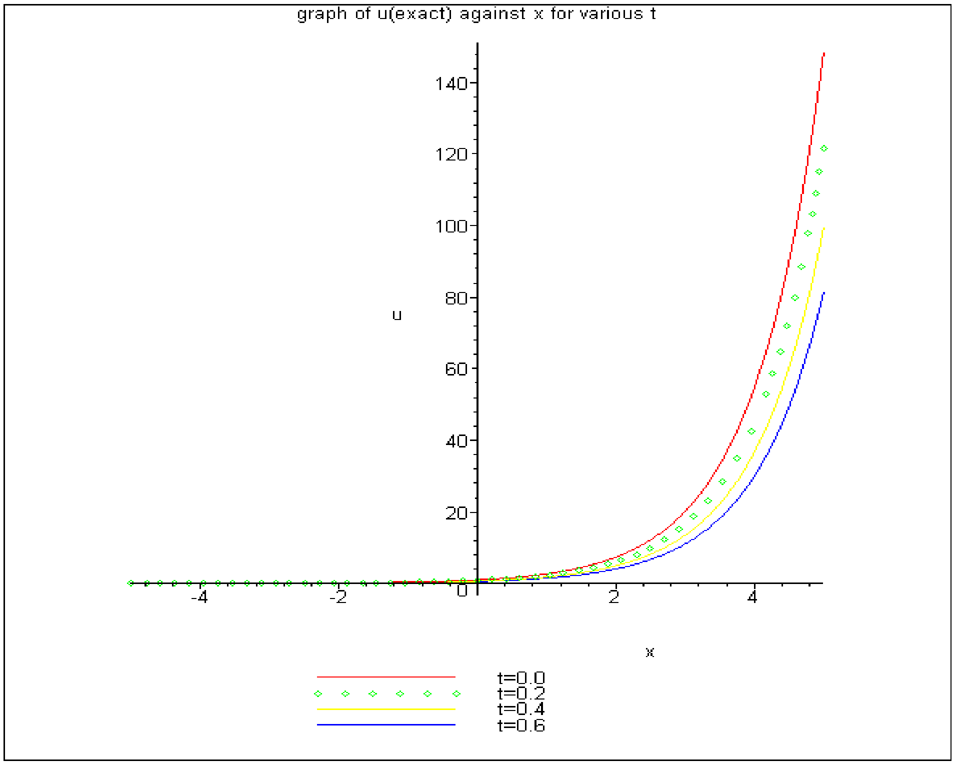

Figure 4

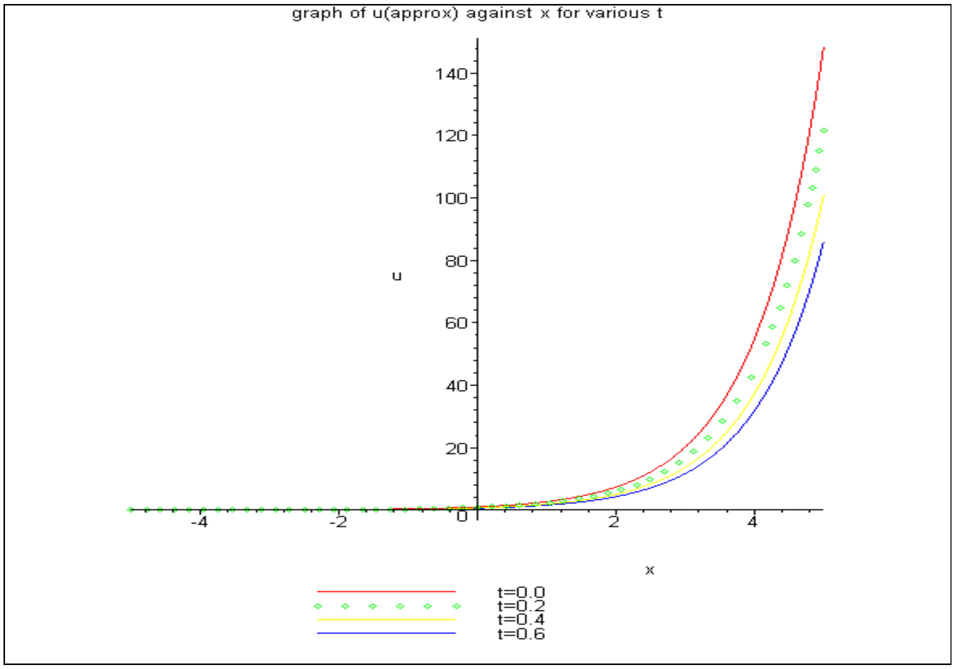

Figure 5

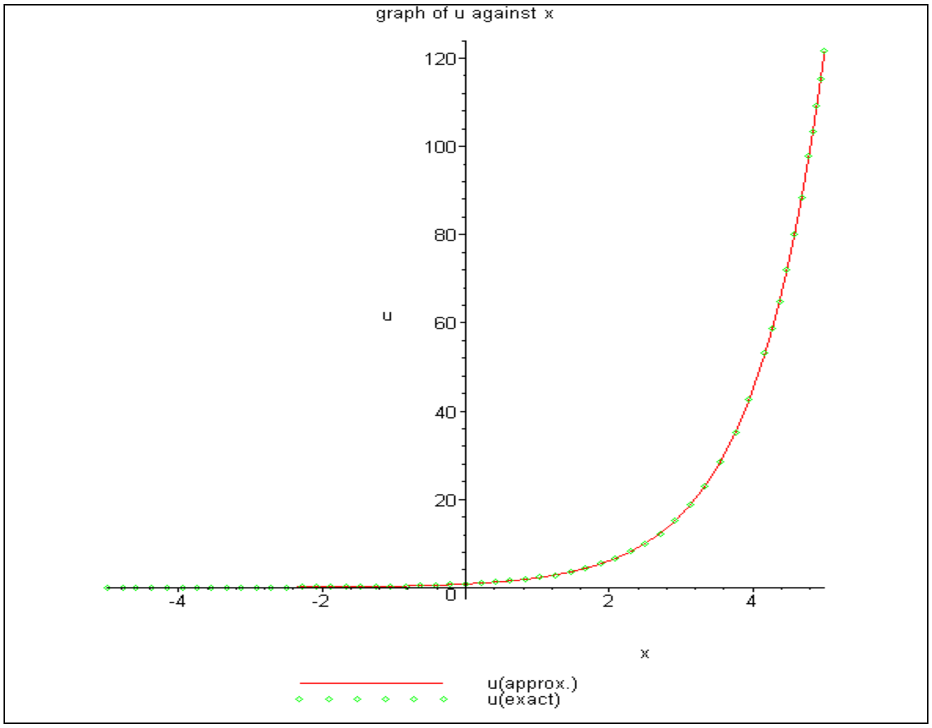

Figure 6

3.3 : Consider a nonlinear non-homogenous partial differential equation ; Dogan (2002)

$$\frac{\partial u}{\partial t} = -e^{u}\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial x^2} + e^{u}(1 + x + t) . \tag{3.13}$$

$$u(x,0) = \ln x , x \neq 0$$

Appling MVIM to (3.13)

$$u_0 = \ln x + \frac{t}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.14)}$$

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda \left[\frac{\partial u_n}{\partial \tau} + e^{u_n}\frac{\partial u_n}{\partial x} - \left(\frac{\partial u_n}{\partial x}\right)^2 - \frac{\partial^2 u_n}{\partial x^2} - e^{u_n}(1+x+t)\right] d\tau \tag{3.15}$$

In the same way, we compute other components for $n = 1, 2 . .$

Figure 7 shows the graphs of exact solution and MVIM solution against x for (3.13).

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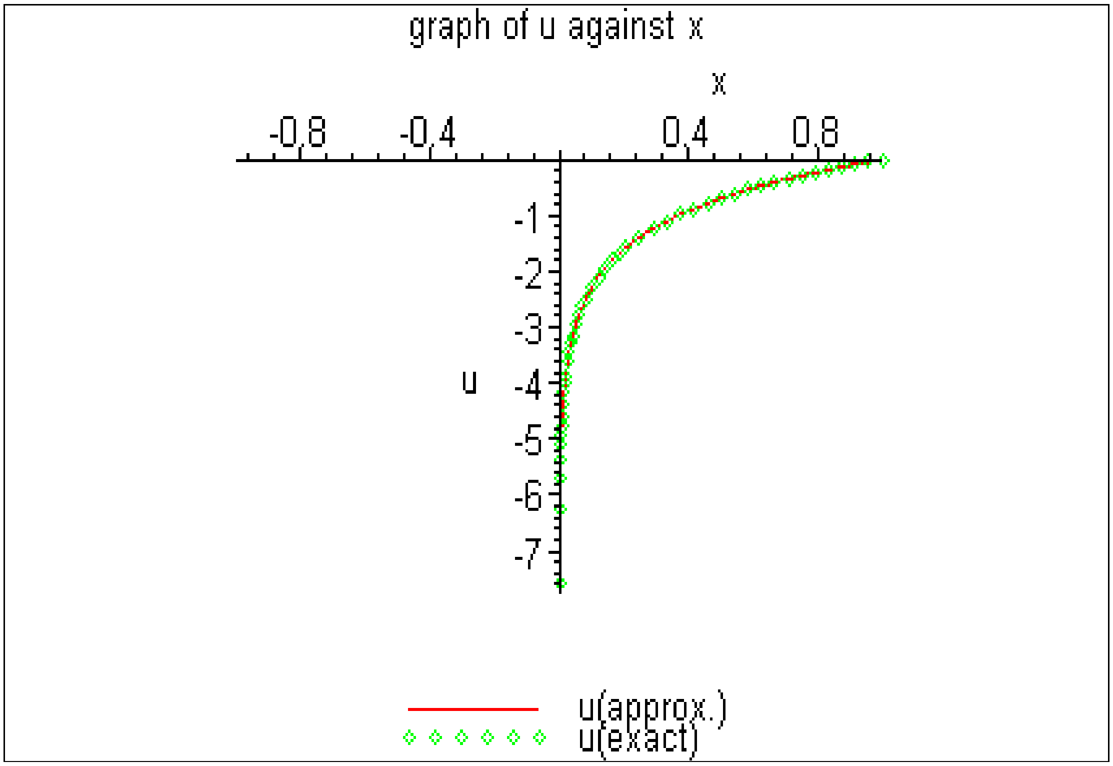

Figure 7

4. CONCLUSIONS

In this work, the MVIM was applied to the solution of nonlinear partial differential equations. The numerical results demonstrated that the method is accurate, reliable and converges faster with less computation when compared with other methods in the literature.

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